

# Belief, Credence, and the Monotonicity Principle

Tazo Tokhadze (University of Sussex)

t.tokhadze@sussex.ac.uk

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*The handouts follow the slides of my PowerPoint presentation. The handouts are slightly more detailed and contain some explanatory remarks.*

## Part 1: Introduction

How does qualitative, all-or-nothing **belief** relate to **degrees of belief** (or credence)?

- Belief is a qualitative attitude towards a proposition: you either believe it will rain tomorrow or don't.
- Credence is a numerical attitude, where you can have various degrees of confidence in a proposition: say, you may be roughly 80% confident that it will rain tomorrow.

**Our focus:** one of the most plausible necessary conditions on how rational belief and credence ought to interact, called the **Monotonicity** principle:

Monotonicity: If an agent believes a proposition,  $X$ , and if she considers another proposition  $Y$  to be at least as probable as  $X$ , then she should also believe  $Y$ .

**Motivation:** Why care about Monotonicity?

- Monotonicity has important implications for what kind of normative theory of the belief-credence relationship is correct.
- Leitgeb's result (2014, 2017): if rational beliefs are deductively cogent and if rational credences satisfy the axioms of probability, then there is only one, **unique theory** that satisfies Monotonicity - Leitgeb's stability theory of belief.<sup>1</sup>

**An outline of the talk:**

**My aims:** to present an argument against Monotonicity and propose an alternative to it.

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<sup>1</sup> Roughly, the stability theory is the view that a rational agent believes a proposition if and only if she assigns a *stably high* degree of belief to the proposition.

Plan: Two main parts

- Part 2: An initial motivation for rejecting Monotonicity: the restrictiveness worry.
- Part 3: An alternative to Monotonicity, **Partial Monotonicity**; Monotonicity can be violated but only with respect to *inferentially trivial disjunctions*.
- A brief discussion of some of the results I've proven by using Partial Monotonicity.
- Conclusion: Partial Monotonicity is a superior alternative to Monotonicity.

## Part 2: Monotonicity is too Restrictive

**The framework:** the agents who have at least two kinds of doxastic attitudes: (qualitative) belief and (numerical) credence.

- We let *Bel* to denote an agent's belief set (a set of all propositions believed by an agent). *Bel*(*X*) means that *X* is believed.
- We assume that *Bel* is **deductively cogent**: *Bel* is consistent; and *Bel* is closed under conjunction: if *Bel*(*X*) and *Bel*(*Y*), then *Bel*(*X* ∧ *Y*).
- We let *P* to denote an agent's credence function. We assume that *P* is a probability function.

Monotonicity in symbols: For all *X* and *Y*, if *Bel*(*X*) and  $P(Y) \geq P(X)$ , then *Bel*(*Y*).

I turn to the initial motivation for rejecting Monotonicity, which is based on the formal result:

**Surprising Result:** Let *Poss*(*X*) mean that *X* is *doxastically possible* from an agent's perspective. Formally, *Poss*(*X*) iff  $\neg Bel(\neg X)$ . Then, if *Bel* is deductively cogent and *P* is a probability function, then Monotonicity is logically equivalent to the following:

- For any propositions *X*, an agent believes *X* if and only if for all doxastically possible propositions *Y*, her credence in *X* given *Y* is high (higher than 50%).
- In symbols: *Bel*(*X*) iff for all *Y* such that *Poss*(*Y*),  $P(X|Y) > 0.5$

## How to interpret Surprising Result?

- $P(X|Y)$  is an agent's estimate of her future probability in *X*, if she learns *Y* and nothing else.
- Given Monotonicity, an agent's belief would be *stable* under learning new information:

- the agent would not consider it to be likely to learn a new piece of information that would make her believed propositions improbable.
- **A Worry:** Too much stability? A scientist may rationally believe a theory, even if it is a real possibility for her to learn new information that would render the theory improbable.

### Counterexample

I'll develop the restrictiveness worry by analysing an example:

Richard believes that Hannes is either a German citizen ( $G$ ) or was born in Austria ( $A$ ), but not both:  $Bel(G \vee A)$ . (where  $\vee$  is the exclusive disjunction,  $G \vee A = (G \wedge \neg A) \vee (\neg G \wedge A)$ ). Richard does not believe anything more specific than  $G \vee A$ .

Now, suppose that Richard's main reason for believing  $G \vee A$  is its first disjunct:  $G \wedge \neg A$ . For this reason, if Richard supposes that its main reason for believing  $G \vee A$  is false, then Richard will no longer have a high confidence in  $G \vee A$  (higher than 0.5).

That is:  $P(G \vee A | \neg(G \wedge \neg A)) < 0.5$ .

This violates Monotonicity. As  $G \wedge \neg A$  is not believed, we have  $Poss(\neg(G \wedge \neg A))$ . And by Monotonicity:  $P(G \vee A | \neg(G \wedge \neg A))$  should be greater than 0.5. Contrary to the example.

**Conclusion:** Monotonicity prohibits believing perfectly reasonable disjunctive beliefs.

But, if Monotonicity is false, what are the alternatives?

### Part 3: An Alternative to Monotonicity

- **Partial Monotonicity:** a logically weaker principle that avoids the restrictiveness worry, but captures some of the important aspects of the original principle.
  - Monotonicity can be violated but only with respect to, what I call, *inferentially trivial disjunctions*.
- A disjunction  $X \vee Y$  is inferentially trivial for an agent when (i)  $X \vee Y$  is logically weaker than at least one of the agent's beliefs, and (ii)  $X \vee Y$  is unreliable in disjunctive syllogism; that is, one cannot reliably use the following rule on  $X \vee Y$ :

$$\frac{X \vee Y \quad \neg Y}{X}$$

- **Example:** Richard believes that Rudolf owns a ford ( $R$ ). But he has no clue on whether Jones owns a ford ( $J$ ). Disjunction  $R \vee J$  is inferentially trivial for Richard.
- $X \vee Y$  is **unreliable** in disjunctive syllogism iff  $P(X \vee Y | \neg Y) \leq 0.5$ .

### Definition of Partial Monotonicity:

**Partial Monotonicity:** For any propositions  $X$  and  $Y$ , if  $Bel(X)$  and  $P(Y) \geq P(X)$ , then  $Bel(Y)$ , *only if  $Y$  is not an inferentially trivial disjunction*.

- Why favour Partial Monotonicity over Monotonicity?

Two overall reasons:

- Avoids the restrictiveness worry; a plausible theory that satisfies Deductive Cogency and Partial Monotonicity but violates Monotonicity. Logically equivalent to Lin and Kelly's (2012, 2021) tracking theory.
- Reduces the number of junk beliefs that an agent is required to have. Cluttering our minds with junk beliefs is bad.

### Concluding Remarks

- A plausible alternative to Monotonicity, Partial Monotonicity: avoids the restrictiveness worry and limits junk beliefs.
- It is a fruitful view: motivates the tracking theory of the belief-credence relationship.
  - Other theories? Maybe. But all the other theories I've studied that satisfy Partial Monotonicity, Deductive Cogency, and Probabilism are logically equivalent to the tracking theory.
- In conclusion: Partial Monotonicity is a superior alternative to Monotonicity.

### References

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